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A Capacity-Based Approach to Receiver Sensitivity for Atmospheric Lasercom Systems

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Abstract: We present an approach to analyzing receiver sensitivity in a fading channel that is rooted in capacity analysis. The approach supports rapid design trades during the early stages of system design.

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1. Introduction

Receiver sensitivity is a critical parameter in free-space lasercom systems because it drives aperture sizes and transmitter powers, and therefore overall system size, weight, and power requirements. The analysis of lasercom receiver sensitivity is complicated by the presence of fading caused by atmospheric turbulence. Forward error-correction (FEC) coding in conjunction with interleaving can alleviate the burst errors associated with atmospheric fading; however, there is still a power penalty relative to the static (non-fading) channel, which we refer to as dynamic loss. The value of the dynamic loss is highly dependent on system design parameters, including the code rate, the code performance, and the interleaver depth. Receiver sensitivity is also affected by other design choices such as the modulation format and receiver architecture.

A capacity-based approach to analyzing receiver sensitivity has been proposed [1] to aid the process of system design. By working explicitly in terms of channel capacity, it becomes clear which aspects of the system are already approaching fundamental limits and which may still be further improved. Thus, initial design trades can often be simplified. The method begins with the fundamental capacity for a static channel with a particular modulation format, receiver architecture, and code rate. From there, penalties are added for the performance of the specific code in the static channel and the dynamic loss, which in turn is comprised of the reduction of capacity in the fading channel, the performance of the selected code in the fading channel, and the effect of residual power correlation between bits caused by a non-infinite interleaver.

We illustrate this capacity-based approach to receiver sensitivity using an optical uplink from the ground to an aircraft with DPSK modulation and a hard decision optically preamplified receiver. In the example, we will use a Reed-Solomon codec whose performance has a closed form expression. We also discuss the Monte Carlo simulations necessary to obtain performance for non-algebraic codes. For our purposes, we neglect hardware implementation losses. The sensitivity approach has proved useful in recent system design and implementation [2].

2. Capacity in Static and Fading Channels

The optical channel capacity $C(N_s)$ is the maximum achievable code rate given N_s average received photons per channel symbol. Expressions for $C(N_s)$ vary depending on the particular optical receiver. For our sensitivity analysis, it is more useful to use capacity to determine the minimum N_s at which error-free communication is possible for a fixed code rate. For a pre-amplified binary hard-decision DPSK receiver, we achieve capacity when the signal has 6.53, 5.20, and 4.81 dB photons per source bit for code rates 0.94, ¾, and ½, respectively.

Atmospheric turbulence causes random fluctuations in the received signal power, affecting the information capacity of the channel. We model the received power in a channel symbol as VN_s, where V is a random variable. In the case where the receiver has perfect channel state information (*i.e.* the instantaneous received power is ideally monitored), the overall faded channel capacity is the expected value of the instantaneous capacity [3]: $C_{fade}(N_s) = E[C(VN_s)]$ The expectation is typically computed numerically.

The random variable V represents the fluctuation of the received power. It is normalized to have unit mean (0 dB), so that it represents both "fades" and "surges." Ideal receiver sensitivity is degraded in a fading environment for most code rates of interest. Lower rate codes typically experience a smaller capacity penalty due to fading. For our example channel, the capacity penalty for fading is 0.95 dB at rate 0.94, is 0.57 dB at rate ¾, and is 0.25 dB at rate ½. Note that the optimum code rate is a function of the channel parameters, i.e., the optimum code rate in the static channel is not necessarily the same as the optimum for a fading channel.

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3. Code Performance in Static and Fading Channels

While capacity calculations bound sensitivity performance, the link budget must reflect the performance of a specific code. Some algebraic codes (e.g., Reed-Solomon and BCH codes) have closed form decoding performance. For many other codes (e.g., LDPC and turbo codes), performance is computed via Monte Carlo simulation. One may be tempted to turn to published coding gains (or distance from capacity) for the selected code. While this is a reasonable short cut, published code performance is often calculated for an additive white Gaussian noise (AWGN) channel which may not accurately reflect the statistics of the receiver in the static channel. As an example, for a BPSK signal with AWGN, the coding gain for the standard telecommunications Reed-Solomon (255,239) code is 5.1 dB (at a BER of 1E-10); these statistics describe the behavior of a coherent heterodyne receiver. For the optically pre-amplified DPSK receiver, where the statistics are chi square [4], the coding gain is only 4.5 dB.

AWGN coding gains also provide little insight into code performance in fading channels. For the lasercom systems of interest, the symbol rate (Mbaud to Gbaud) is much faster than the fading coherence times (~1-20 ms); thus, the signal strength for consecutive channel symbols is highly correlated. Without compensation, this correlation persists longer than the size of a codeword, allowing a single fade 'event' to cause one or more decoding block errors. The standard remedy is to employ an interleaver, thereby separating the symbols of a codeword in time. For sufficiently deep interleavers where the codeword symbols are all separated by more than the fading coherence time, we can model each codeword symbol as statistically independent. In effect, we model the interleaver as infinite, significantly simplifying the computation of code performance. We model each symbol with two random variables: one representing the signal level, and the other representing the noise.

Consider our example case of an optically preamplified binary DPSK receiver. For a static channel, the probability of bit error is given [4] as $p_e = 0.5\times\exp(-N_s)$. An $[n,k]$ Reed-Solomon code corrects $t=(n-k)/2$ code symbol errors, where each symbol represents m bits. If any of the bits are in error, the symbol is in error. For best performance, interleaving should be done so that all of the symbol bits arise from the same "fade" or "surge" random variable V. For an m-bit symbol, the symbol error is thus calculated as

$$p_{SymError}(N_S) = 1 - \int dv (1 - p_e(vN_S))^m f_V(v)\,. \tag{1}$$

Since each symbol is independent, the total number of symbol errors in the code word is a Binomial random variable. The block is decoded incorrectly if more than t errors occur.

4. Finite Interleaver

Code performance modeling for a finite interleaver is challenging, whether we pursue algebraic error-counting or Monte Carlo analysis. The atmospheric scintillation is a random process; after interleaving, the codeword symbols' received signal levels are non-trivially correlated. To be completely precise with such a model, we must draw these random variables from a joint probability distribution function, which quickly becomes computationally unwieldy. In order to avoid this, we use a block fading model. The received power is modeled as a sequence of independent random variables, each representing the power for a time of duration T_c, the coherence time of the fading process. As a result, the received power levels for two symbols are either perfectly correlated or completely uncorrelated.

The finite interleaver computation for a block fading model is straightforward for Monte Carlo simulation. As in the infinite interleaver case, we generate two random numbers, one representing the signal level and the other representing the noise level. For a finite interleaver, the signal-level random variable is used for consecutive received bits, while each bit uses an independent noise random variable.

The finite interleaver computation is more complicated for the algebraic cases, such as Reed-Solomon codes. We define the random variable K_i as the number of symbol errors arising from the fade V_i. When M symbols are drawn from the same random signal level, we calculate the probability that k of them are in error as

$$\Pr(K_i = k) = \binom{M}{k} \int p_{sym}^k(vN_S)(1 - p_{sym}(vN_S))^{M-k} f_V(v)dv\,. \tag{2}$$

The total number of symbol errors in the code block is the sum of the independent random variables K_i, whose probability mass function can be computed by successive convolution. As before, the block is decoded correctly for less than t errors.

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5. Atmospheric Channel Example and Discussion

We illustrate the method using an atmospheric channel based on the Hufnagel-Valley 5/7 C_n^2 model [5]. The ground transmitter has an 11-mm aperture and an elevation angle of twenty degrees. The aircraft has a 22-mm receive aperture and is flying perpendicular to the beam at 75 knots (38.6 m/s) and 30 kft, corresponding to a link range of 26.6 km. The channel was modeled via a wave-optics simulation, which incorporated correction of atmospheric tilt on the uplink beam by a fast steering mirror in the ground terminal with a 1-kHz tracking loop using a downlink beam from the aircraft as a tracking reference. The resulting scintillation index is 0.155, corresponding to moderate turbulence. The time series is normalized to the mean value, and the average reduction in intensity due to beam spreading is treated as a channel loss (0.36 dB) in the link budget. The probability density function of the time series data was fit to a gamma-gamma distribution [6], producing a fit of (α, β) = (12.70, 12.67). The coherence time T_c was determined to be 8.2 ms by autocorrelation of the time series.

Figure 1 shows the performance of rate 0.94, ¾, and ½ Reed-Solomon codes in the above atmospheric channel. The receiver sensitivity budgets, shown in Table 1, illuminate system trade-offs. The capacity terms illustrate the benefit of the lower rate codes, with the best capacity of the three at rate ½. The Reed-Solomon code, however, does not achieve the full capacity benefit; the additional hardware complexity of the rate ½ code does not improve performance relative to the rate ¾ code. We also see clearly the potential for improvement with the selection of a more advanced code capable of achieving performance closer to capacity. Finally, another clear conclusion is that in this example, a deeper interleaver would not provide significant performance enhancement.

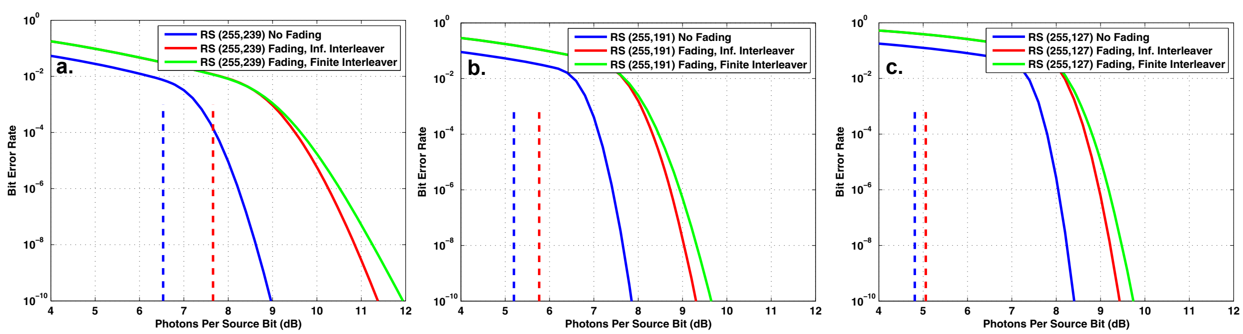

Fig. 1. Sample receiver sensitivity calculations for (a) R = 0.94 [255,239], (b) R = ¾ [255,191], and (c) R = ½ [255,127]. The dashed lines represent capacity in static (blue) and fading (red) channels. The finite interleaver assumes three code symbols per block fade. The distance between the blue and green solid curves is the dynamic loss.

Table 1: DPSK Receiver Sensitivity Budgets for R = 0.94, ¾, and ½ at BER=10^{-10}

Code Rate	**Rate 0.94**	**Rate 3/4**	**Rate 1/2**
Sensitivity at capacity (static channel)	6.53 dB ppb	5.20 dB ppb	4.81 dB ppb
Code gap to capacity (static channel)	2.43 dB	2.65 dB	3.59 dB
Dynamic loss	2.98 dB	1.80 dB	1.33 dB
Reduction in capacity	*1.13 dB*	*0.57 dB*	*0.25 dB*
Code penalty for fading (infinite interleaver)	*1.28 dB*	*0.88 dB*	*0.77 dB*
Interleaver implementation loss	*0.57 dB*	*0.34 dB*	*0.31 dB*
Ideal Sensitivity in Fading Channel	**11.94 dB ppb**	**9.65 dB ppb**	**9.74 dB ppb**

6. References

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